**(LP Algebraic Formulation – 20 Points)**

A large paper manufacturing company has **10 mills** from which it needs to supply **1,000 customers**. It uses **three alternative types of machines** and **four types of raw materials** to make **five different types of paper**. The company needs to develop a detailed **production-distribution plan** for the upcoming **month**, with the objective of **minimizing the total cost** of producing and distributing the paper during the month. Specifically, it is necessary to determine the **amount of each type of paper** to be made at **each paper mill** on each **type of machine** *and* the **amount** of **each type of paper to be shipped** from each **paper mill** to each **customer**.

The relevant data can be expressed symbolically as follows:

*Djk* = number of units of paper type *k* demanded by customer *j*.

*rklm*= number of units of raw material *m* needed to produce 1 unit of paper type *k* on machine type *l*.

*Rim* = number of units of raw material *m* available at paper mill *i*.

*ckl* = number of capacity units of machine type *l* that will produce 1 unit of paper type *k*.

*Cil* = number of capacity units of machine type *l* available at paper mill *i*.

*pikl* = production cost for each unit of paper type *k* produced on machine type *l* at paper mill *i*.

*tijk* = transportation cost for each unit of paper type *k* shipped from mill *i* to customer *j*.

(a) Using the above notation, provide a **complete algebraic formulation**. Make sure to clearly define the:

* Indices
  + I = Set of Paper Mills
  + i = paper mill (1,2,…,10)
  + J = Set of Customers
  + j = customer (1,2,…,1,000)
  + K = Set of Paper Types
  + k = paper type (1,2,…,5)
  + L = Set of Machine Types
  + l = machine type (1,2,3)
  + M = Set of Raw Materials
  + m = raw material (1,2,…,4)
* Parameters
  + Djk = number of units of paper type k demanded by customer j.
  + rklm= number of units of raw material m needed to produce 1 unit of paper type k on machine type l.
  + Rim = number of units of raw material m available at paper mill i.
  + ckl = number of capacity units of machine type l that will produce 1 unit of paper type k.
  + Cil = number of capacity units of machine type l available at paper mill i.
  + pikl = production cost for each unit of paper type k produced on machine type l at paper mill i.
  + tijk = transportation cost for each unit of paper type *k* shipped from mill *i* to customer *j*.
* Decision variables, (4 points)
  + Xi,k,l = Amount of each Type of Paper k, to be made at each Paper Mill i, on each type of Machine l.
  + Yi,j,k = Amount of each Type of Paper k, to be shipped from each Paper Mill i, to each customer j.
* Constraints, and (10 points)
  + Raw Materials m available at paper mill i
    - Sum(kl)(rk,l,m\*xi,k,l) <= Ri,m
  + Machine l Capacity for paper type k
    - Sum(i)(xikl)<=ckl
  + Machine l Capacity at paper mill i
    - Sum(k)(xikl) <= Cil
  + Demand
    - sum(i,l))(xikl) = sum(j)(Djk)
  + Xi,k,l >= 0, A i in I, k in K, l in L
  + Yi,j,k >= 0, A k in K, i in I, j in J
* Objective function. (3 points)
  + Minimize the total cost of producing and distributing the paper during the month; Production Cost + Transportation Cost
  + Min Sum(ikl)(pikl\*Xikl)+sum(ijk)(tijk\*Yijk)

(b) Count the number of the actual decision variables and constraints by type for the model. (3 points)

**Question 2 (AMPL and Excel Implementations – 40 Points)**

(NOTE: For this question, you will be graded on both the *quality* and *validity* of your AMPL and Excel implementations).

Green Earth is an organization that operates a reclamation center that collects four types of solid waste materials and treats them so that they can be amalgamated into a salable product. Note that treating and amalgamating are separate processes. Three different grades of this product can be made, depending on the mix of materials used (see first column in Table 2a). Although there is some flexibility in the mix for each grade, quality standards may specify the minimum or maximum weight allowed for the proportion of a material in the product grade, whereby the proportion is the weight of the material expressed as percentage of the total weight for the product grade. For each of the two higher grades, a fixed percentage is specified for one of the materials. These specifications are given in Table 2a along with the cost of amalgamation and the selling price for each grade.





The reclamation center collects its solid waste materials from regular sources and so is normally able to maintain a steady rate of such materials for treating them. Table 2b gives the quantities available for collection and treatment each week, as well as the cost of treatment, for each type of material. In addition, Good Earth has allocated $30,000 per week to cover the entire treatment cost for the solid waste materials. These funds come from contribution and grants raised by the organization. The organization’s mandate requires that at least half the amount available of each material to be actually collected and treated.

The organization’s management wants to determine the amount of each product grade to produce and the exact mix of materials to be used for each grade. The objective is to maximize the net weekly income (total sales minus total amalgamation cost), exclusive of the fixed treatment cost of $30,000 per week that is being covered by gifts and grants. To achieve this, the following LP was formulated.